

Utility-Theoretic Underwriting

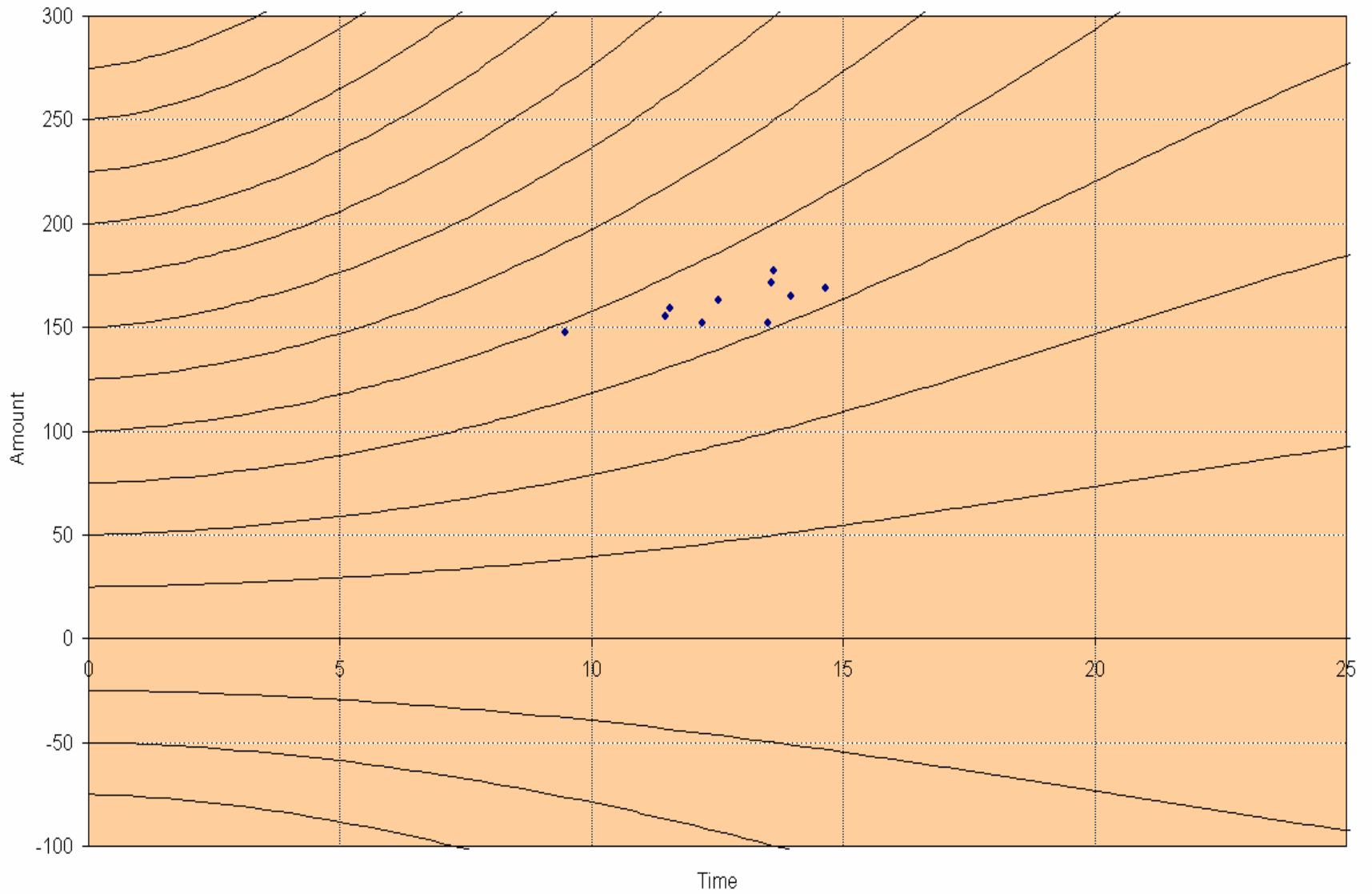
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Outline

- The Four Principles of Valuing Stochastic Cash Flows (Underwriting)
 - Present Value as a Random Variable
 - The Imposture of Risk-Adjusted Discounting
 - Expected Utility and Preferences of Stochastic Wealth
 - Comments on Alternative Theories
 - Optimizing Expected Utility given Price
 - Market Price as Optimizing Everyone's Expected Utility
 - Capital Markets as Information Providers?
- Examples in Excel (Utility-Theoretic Underwriting.xls)
 - Coin Toss for \$100,000
 - Insurance against a Lawsuit
 - Simple Reinsurance Market
 - Capital-Consumption Example

Stochastic Cash Flow on Present-Value Coordinate System



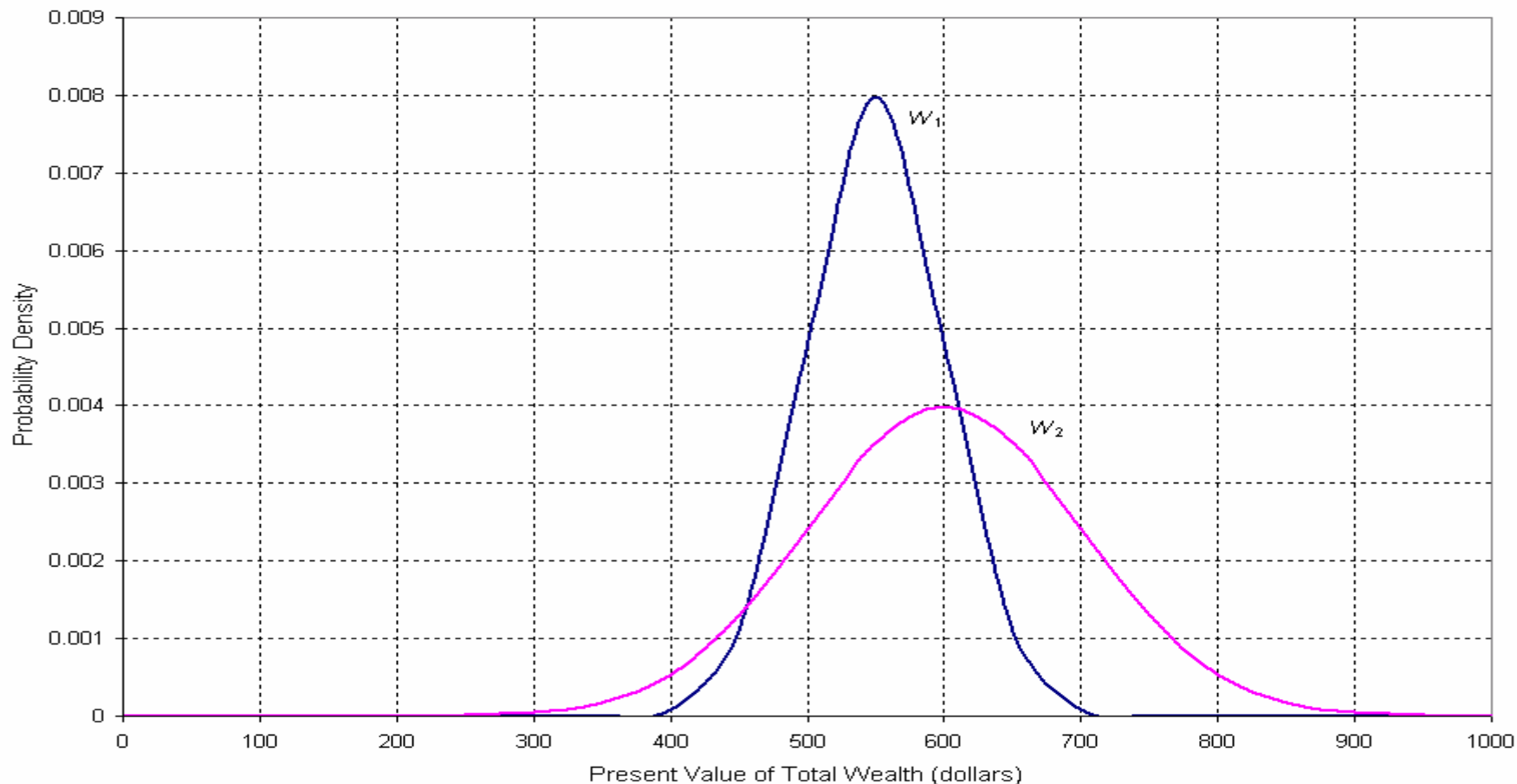
Implications of Present-Value Coordinates

- Present value is a random variable. Exhibit 2 (VSCF, 40) graphs the CDF of the PV of this stochastic cash flow.
- The value of this stochastic cash flow should lie between 50 and 100. If all points lay on the same isobar, the value would be that of the isobar. Risk-adjusted discounting can break out of the envelope.
 - Objection: What if the coordinate system changes?
- If two PV random variables are (almost surely) equal, then they must have the same value. In symbols:
$$\text{Prob}\{PV[X] = PV[Y]\} = 1 \quad \Rightarrow \quad \text{Value}[X] = \text{Value}[Y]$$
- PVs of outcomes are like sufficient statistics. Negative PVs as legitimate as positive; solvency and bankruptcy are constraints, not criteria (footnotes 25 and 26).

Just Say “No!” to Capital Allocation

- Capital (or money) does not work, despite advertising slogans.
- Would capital allocated to a one-year hurricane treaty work off season (Dec-May)? Can we make capital moonlight? Some try to have it both ways.
- Capital allocation inevitably confuses rate of return (% per year) with return (%).
- Example of short-lived exposure:
25% per year is 0.5% per week

Question: Which Wealth is Better, $\$550 \pm 50$ or $\$600 \pm 100$?



Answer: Which is greater, $E[u(W_1)]$ or $E[u(W_2)]$?

Ordering Projects by Expected Utility

- Current stochastic wealth W
Project has present value X and cost q .
Wealth with project is $W+X-q$.
Compare $E[u(W+X-q)]$ with $E[u(W)]$.
- Borch's insurance formulation, $W-L+p$, equivalent (VSCF, 16)
- Instantaneous formulation $\text{Value}=H[X]$ frequent in our actuarial literature.
(e.g., Bühlmann, Gerber)

Some are so future-oriented that they must displace instantaneous results to the end of an accounting period (with interest)! Why is a future time better than the present? Is an instantaneous problem illegitimate?

Comments on Alternatives

Capital Consumption Pricing Example

a)	b)	c)	d)	e)	f)
Premium	Expenses	Probability of Loss	NPV Amount of Loss	(a-b-d) Loss or Profit	Adjusted Amounts
\$1,000,000	\$175,000	20%	\$0	\$825,000	
		15%	\$100,000	\$725,000	
		15%	\$250,000	\$575,000	
		15%	\$350,000	\$475,000	
		10%	\$500,000	\$325,000	
		10%	\$750,000	\$75,000	
		10%	\$1,000,000	-\$175,000	-\$350,000
		2.5%	\$1,500,000	-\$675,000	-\$1,350,000
		2.5%	\$2,500,000	-\$1,675,000	-\$3,350,000
				\$395,000	-\$1,016,667
				Penalty Charge	200.0%
				Mean	\$395,000
				Risk-Adjusted Mean	\$318,750

Downside (Capital Consumed) Amounts Increased (points to d)

Penalty Charge (points to e)

- “NPV Amount of Loss”: Instantaneous approach; no risk-adjusted discounting
- Broken-line utility function?
- CC-adjusted $\mu_{\text{profit}} = \$318,750$; \therefore is fair premium \$681,250? No, it's \$739,000.

Wang Pricing Transform Modifies the Probabilities

Premium	Expenses	Probability of Loss	NPV Amount of Loss	(a-b-d) Loss or Profit	Downside	Upside
\$1,000,000	\$175,000	20%	\$0	\$825,000		\$825,000
		15%	\$100,000	\$725,000		\$725,000
		15%	\$250,000	\$575,000		\$575,000
		15%	\$350,000	\$475,000		\$475,000
		10%	\$500,000	\$325,000		\$325,000
		10%	\$750,000	\$75,000		\$75,000
		10%	\$1,000,000	-\$175,000	-\$175,000	
		2.5%	\$1,500,000	-\$675,000	-\$675,000	
		2.5%	\$2,500,000	-\$1,675,000	-\$1,675,000	
				\$395,000	-\$508,333	\$554,412

Applies a Greater Weight to Downside ... By Modifying Probabilities

Premium	Expenses	Probability of Loss	Cumulative Probability	Lambda	NORMSINV	Transform	Adjusted Probability	Implied Prob	NPV Amount of Loss	Loss or Profit
\$1,000,000	\$175,000	20.0%	20.0%	0.75	(0.84)	(1.59)	5.6%	5.6%	\$0	\$825,000
		15.0%	35.0%		(0.39)	(1.14)	12.8%	7.2%	\$100,000	\$725,000
		15.0%	50.0%			(0.75)	22.7%	9.9%	\$250,000	\$575,000
		15.0%	65.0%			0.39	35.8%	13.1%	\$350,000	\$475,000
		10.0%	75.0%			(0.08)	47.0%	11.2%	\$500,000	\$325,000
		10.0%	85.0%			1.04	61.3%	14.3%	\$750,000	\$75,000
		10.0%	95.0%			1.64	81.5%	20.2%	\$1,000,000	-\$175,000
		2.5%	97.5%			1.96	89.7%	7.2%	\$1,500,000	-\$675,000
		2.5%	100.0%				100.0%	11.3%	\$2,500,000	-\$1,675,000
									Exp Value -- Unadjusted	\$395,000
									Exp Value -- Adjusted	-\$9,100

Target adjusted ENPV (points to -\$9,100)

- WPT-adjusted $\mu_{\text{profit}} = (\$9,099)$; Yes, fair premium is \$1,009,099?
- Would half the deal cost half the price?

New Perspective: How much to buy at a given price?

- Comparing $E[u(W+X-q)]$ with $E[u(W)]$ can give you only a ceiling price (for one unit of X), not the appropriate price.
- An economic agent should be free to choose how much of X to purchase at unit price q . Cash flows are scaleable.
- New objective: Maximize $f(\theta)=E[u(W+\theta X-\theta q)]$. State-price form: $q = E[\Psi X]$.
- “Fundamental Theorem” of Appendix B: For a given q , $f(\theta)$ has one and only one maximum. The curve looks like an upside-down parabola. (cf. VSCF, Exhibit 5, p. 43)
- Two or more agents together find the unique price q at which each maximizes its expected utility and all of X clears. This is a Pareto optimum.
- The agents constitute a market, but each agent is entitled to its own beliefs. The market is an epiphenomenon.

Exponential Utility

$$u(x) = (1 - e^{-ax})/a, \quad u'(x) = e^{-ax}$$

- Just about the only game in town, as argued by Hans Gerber and in VSCF, Appendix C. Has all the desirable properties, including absolute risk aversion (ARA)
- Some argue for power-curve utility and relative risk aversion (RRA).
 - RRA is appropriate for bundles of physical goods, e.g, apples and oranges, which come in non-negative amounts.
 - With SCFs we are dealing with one unit (dollars) in random outcomes that can be positive or negative.
 - Only exponential utility is defined for all real numbers.
 - Only exponential utility allows an X independent of W to be valued by itself. Otherwise, one might have to know everything in order to value anything.

Notes on Excel Examples

- Ex 1: Normal approximations (analytic solution) are very good.
Overall $a = 3.33E-06$ is harmonic sum of individuals. Market persona exists because every has the same risk assessment, but market derives from individuals
Interpretation of a “zero- β ” stochastic cash flow
- Ex 2: Counter intuitively, all risk is insured for $E[X]$.
Insurer disagrees over lawsuit probability; yet Pareto optimum achieved.
Try the example with risk-neutral insurer.
Is the pooling theory of insurance valid?
- Ex 3: Insurer will pay any price at which reinsurers will sign for 100%.
Reins A and B agree on $E[X]$; but B has correlated exposure. Rein C is pessimistic about the risk, estimating twice the pure premium.
Part A: A and B rashly assume 100% for \$5.1 million; C on sidelines at zero.
Part B: C allowed a short position, and 100% clears for \$6.1 million. Market stabilizes; everyone happy except for the insurer.

Quotable Quotes

- [This theory] sets the agents to the virtuous task of extracting value from projects, rather than from one another. (“Valuation of Stochastic Cash Flows,” 2)
- Risk-adjusted discounting has misled many to elevate [solvency] from the status of a constraint to that of a valuation method. (VSCF, 31)
- One should make sound economic decisions and let the accounting chips fall where they may. (33, footnote 18)
- The business of insurance should be to underwrite well, not to underwrite to generate funds to invest well. ... In companies that understand this theory and the near idealness of its application to insurance chief actuaries will be kings. (34)
- “Asking a valuation formula to depend on [wealth level] is like asking a shopkeeper to charge lower prices to the poor than to the rich.” (63)
- To him whose only tool is a hammer everything looks like a nail. (anonymous)
- If I have seen farther, it is by standing on the shoulders of giants. (Isaac Newton)

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